MA 261 **Exam 2** **Summer 2016**

Student's Name: KEY

Student's ID Number: ____________________

Instructions:

1. **Do NOT turn this page until told to do so.**
2. Show ALL work. No credit will be given without proper supporting work. For free response questions, write your final answer in the box provided. For multiple choice questions, clearly circle one answer.
3. There are 10 questions on 8 pages. Once you are allowed to turn the page, check that you have all pages.
4. No electronic devices, books, or notes are allowed. Please turn off your cell phone and put it away.
5. You will have 60 minutes to complete the exam.
6. **Keep your eyes on your own exam, and try to cover your work.**
7. Read the following statement and sign your name below.

Purdue University faculty and students commit themselves towards maintaining a culture of academic integrity and honesty. The students taking this exam are not allowed to seek or obtain any kind of help from anyone to answer questions on this test. If you have questions, consult only an instructor or a proctor. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you finish your exam and hand it in to a proctor or to an instructor. You may not consult notes, books, calculators, cameras, or any kind of communications devices until after you finish your exam and hand it in to a proctor or to an instructor. If you violate these instructions you will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students. Your instructor and proctors will do everything they can to stop and prevent academic dishonesty during this exam. If you see someone breaking these rules during the exam, please report it to the proctor or to your instructor immediately.

I have read and agree to these terms and conditions.

Signature ____________________________________

1. *(8 points)* What is the maximum value of the directional derivative $D_{\mathbf{u}}f(x,y,z)$ at the point $(3,3,0)$ if $f(x,y,z) = \dfrac{x^2e^{xz}}{y}$?

(a) $2\sqrt{11}$
(b) $2\sqrt{19}$
(c) $\sqrt{86}$
(d) $\sqrt{89}$
(e) 10

Max of $D_u f(x,y,z) = |\nabla f(x,y,z)|$

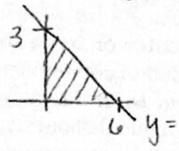
$$\nabla f = \left\langle \frac{2xe^{xz}}{y} + \frac{x^2 z e^{xz}}{y},\ \frac{-x^2e^{xz}}{y^2},\ \frac{x^3e^{xz}}{y} \right\rangle$$

$$\nabla f(3,3,0) = \langle 2, -1, 9 \rangle$$

$$|\nabla f(3,3,0)| = \sqrt{4+1+81} = \sqrt{86}$$

2. *(8 points)* Find M_x, the moment about the x-axis, of the triangular lamina enclosed by the lines $x = 0$, $y = 0$, and $x + 2y = 6$ if the density function, $\rho(x,y)$, is proportional to x.

(a) $3k/4$
(b) $3k$
(c) $27k/2$
(d) $18k$
(e) $54k$



$y = 3 - \frac{x}{2}$

$$M_x = \iint_D y\rho(x,y)\,dA$$

$$= \int_0^6 \int_0^{3-\frac{x}{2}} kxy\,dy\,dx$$

$$= \int_0^6 \frac{k}{2}x\left(3-\frac{x}{2}\right)^2 dx$$

$$= \int_0^6 \frac{k}{2}\left[9x - 3x^2 + \frac{x^3}{4}\right] dx$$

$$= \frac{k}{2}\left[\frac{9}{2}x^2 - x^3 + \frac{x^4}{16}\right]_0^6$$

$$= \frac{k}{2}\left[27(6) - 36(6) + 81\right]$$

$$= k\left[-27 + \frac{81}{2}\right] = \frac{27k}{2}$$

3. *(8 points)* Evaluate $\int_0^2 \int_{y^2/4}^1 ye^{x^2}\,dx\,dy$ by changing the order of integration.

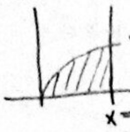
(a) 0

(b) $e - 1$

(c) $2e$

(d) $3e^2 + 2$

(e) $e^4 - 1$



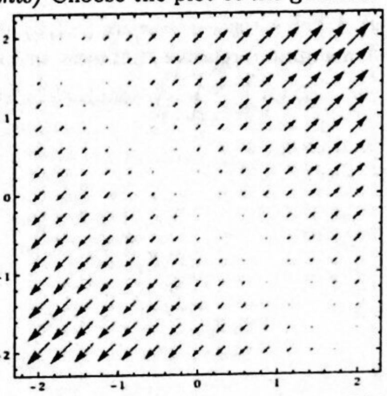
$x = y^2/4 \rightarrow y = 2\sqrt{x}$

$x = 1$

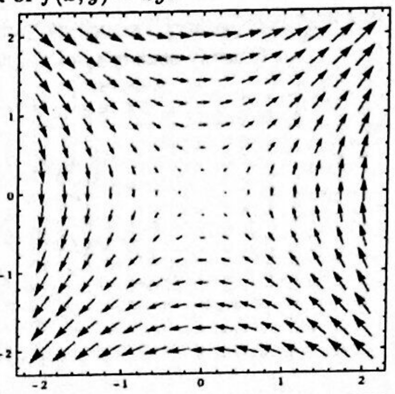
$$\int_0^1 \int_0^{2\sqrt{x}} ye^{x^2}\,dy\,dx = \int_0^1 2xe^{x^2}\,dx \overset{u=x^2}{=} e^{x^2}\Big|_0^1 = e - 1$$

4. *(8 points)* Choose the plot of the gradient vector field of $f(x,y) = xy$. $\longrightarrow \nabla f = \langle y, x\rangle$

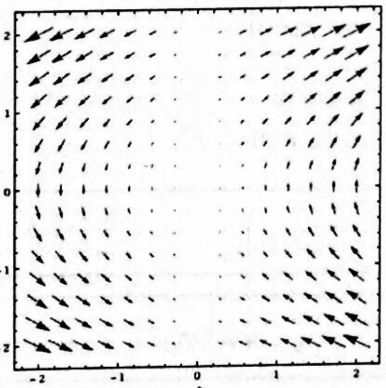
$\langle 0,0\rangle$ on $y=-x$



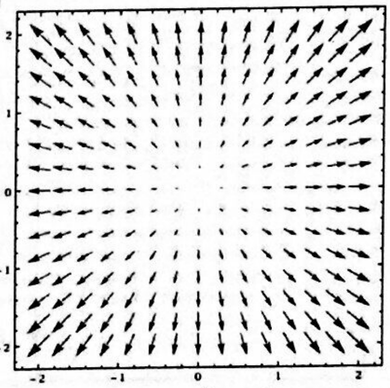
(a)



(c)



(b)



(d)

$\langle 0,0\rangle$ on $x=0$

x - component positive for $x>0$

5. *(14 points total)*

(a) *(7 points)* Find the critical point(s) of $f(x, y) = x^3 + 6x^2 + 3y^2 + 9x$.

$f_x = 3x^2 + 12x + 9 \overset{set}{=} 0 \longrightarrow 3(x+3)(x+1) = 0$

$x = -1, -3$

$f_y = 6y \overset{set}{=} 0 \longrightarrow y = 0$

$\boxed{(-1,0)\ ,\ (-3,0)}$

(b) *(7 points)* The critical points of $f(x, y) = \frac{1}{3}x^3 - 2xy + y^2 - x$ are $(-1, 0)$ and $(2, 3/2)$. Classify each critical point as a relative maximum, relative minimum, or saddle point.

$f_x = x^2 - 2y - 1$

$f_y = -2x + 2y$

$f_{xx} = 2x$

$f_{yy} = 2$

$f_{xy} = -2$

$D = 4x - (-2)^2 = 4x - 4$

	D	f_{xx}	
$(-1,0)$	$-8<0$		saddle
$(2, 3/2)$	$4>0$	$4>0$	rel min

(-1,0): saddle

(2,3/2): rel. min.

6. *(10 points)* **Find the maximum value of $f(x,y) = 3x^2 + y^2 - 4x$ along the boundary of the region $\{(x,y) | x^2 + y^2 \leq 4\}$ by using Lagrange Multipliers.**

$$\begin{cases} ① & 6x - 4 = 2\lambda x \\ ② & 2y = 2\lambda y \to y = 0 \text{ or } \lambda = 1 \\ ③ & x^2 + y^2 = 4 \end{cases}$$

$\lambda = 1$: ① $6x - 4 = 2x \to x = 1$

③ $1^2 + y^2 = 4 \to y = \pm\sqrt{3}$

$y = 0$: ③ $x^2 = 4 \to x = \pm 2$

(x,y)	$f(x,y)$
$(1, \pm\sqrt{3})$	2
$(2,0)$	4
$(-2,0)$	20 ← max

$\boxed{20}$

7. *(4 points)* **Evaluate** $\int_1^3 \int_1^{e^{3x}} \frac{x}{y}\, dy\, dx.$ $= \int_1^3 x \ln y \Big|_1^{e^{3x}} dx$

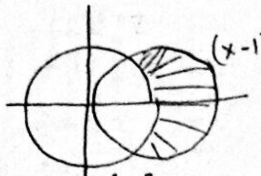
$= \int_1^3 3x^2 dx$

$= x^3 \Big|_1^3 = 26$

$\boxed{26}$

8. ***(14 points total)*** **Sketch the region D, and *set up* (but do not evaluate) an iterated integral to find the following:**

(a) ***(7 points)*** $\int\int_D x\,dA$, where D is the region in the xy-plane inside the circle $(x-1)^2+y^2=1$ and outside the circle $x^2+y^2=1$.



$(x-1)^2+y^2=1 \to x^2-2x+y^2=0$

$\to r^2=2r\cos\theta$

$\to r=2\cos\theta$

$x^2+y^2=1 \to r=\pm 1$

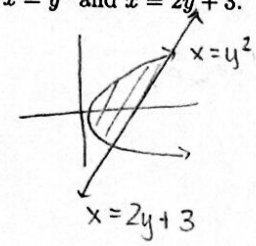
$2\cos\theta=1$

$\cos\theta=1/2$

$\theta=\frac{\pi}{3}, -\frac{\pi}{3}$

$$\boxed{\int_{-\pi/3}^{\pi/3}\int_{1}^{2\cos\theta} r\cos\theta \cdot r\,dr\,d\theta}$$

(b) ***(7 points)*** $\int\int_D x\,dA$ where D is the region in the xy-plane bounded by the curves $x=y^2$ and $x=2y+3$.



$y^2=2y+3$

$y^2-2y-3=0$

$(y-3)(y+1)=0$

$y=-1, 3$

$$\boxed{\int_{-1}^{3}\int_{y^2}^{2y+3} x\,dx\,dy}$$

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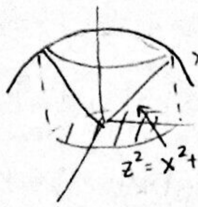
9. ***(16 points total)*** ***Set up*** **the integral $\int\int\int_E x\,dV$, where E is the solid above the cone $z^2 = x^2 + y^2$ and below the sphere $x^2 + y^2 + z^2 = 8$ using:**

(a) ***(8 points)*** **cylindrical coordinates**



$x^2+y^2+z^2=8$
$\rightarrow z=\sqrt{8-r^2}$

$z^2=x^2+y^2 \rightarrow z=r$

$x^2+y^2+(x^2+y^2)=8$
$x^2+y^2=4$
$r=2$

$x=r\cos\theta$
$dV = r\,dr\,d\theta$

$$\boxed{\int_0^{2\pi}\int_0^{2}\int_r^{\sqrt{8-r^2}} r\cos\theta \cdot r\,dz\,dr\,d\theta}$$

(b) ***(8 points)*** **spherical coordinates**

$z^2=x^2+y^2$
$\rho^2\cos^2\phi=\rho^2\sin^2\phi$
$\tan^2\phi=1$
$\phi=\frac{\pi}{4}, \cancel{\frac{3\pi}{4}}$

$x^2+y^2+z^2=8$
$\rho^2=8$
$\rho=\sqrt{8}$

$x=\rho\sin\phi\cos\theta$
$dV=\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$

$$\boxed{\int_0^{\pi/4}\int_0^{2\pi}\int_0^{\sqrt{8}} \rho\sin\phi\cos\theta\cdot\rho^2\sin\phi\,d\rho\,d\theta\,d\phi}$$

10. *(10 points total)*

(a) *(4 points)* Write parametric equations to represent the lower half of the unit circle $x^2 + y^2 = 1$ (include the bounds on the parameter).

Some possible answers:

① $x = \cos t,\ y = -\sin t,\ 0 \le t \le \pi$

② $x = \cos t,\ y = \sin t,\ \pi \le t \le 2\pi$

③ $x = t,\ y = -\sqrt{1-t^2},\ -1 \le t \le 1$

(b) *(6 points)* Evaluate the line integral $\int_C x^2 z\, ds$ where C is the curve with parametric equations $x = 2\sin t, y = t, z = -2\cos t, 0 \le t \le \pi$.

$$\int_C x^2 z\, ds = \int_0^{\pi} (2\sin t)^2(-2\cos t)\underbrace{\sqrt{(2\cos t)^2 + 1^2 + (2\sin t)^2}}_{\sqrt{5}}\, dt$$

$$= -8\sqrt{5}\int_0^{\pi} \sin^2 t \cos t\, dt$$

$u = \sin t$

$$= -8\sqrt{5}\left(\tfrac{1}{3}\sin^3 t\right)\Big|_0^{\pi}$$

0